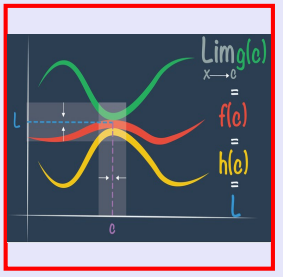
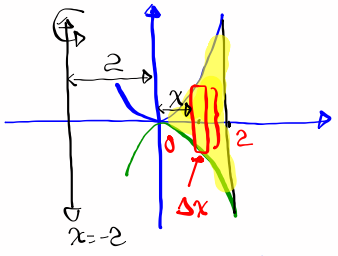
Calculus I

Lecture 57



Feb 19-8:47 AM

Draw the region bounded by $f(x)=x^2$, $g(x)=-x^2$, and $x=2$.



1) find its area.

$$A=\int_0^2 \text{Top}-\text{Bot}\,dx$$

$$=\int_0^2 [x^2-(-x^2)]=\int_0^2 2x^2\,dx=\square$$

2) Suppose we rotate this region by $x=-2$, find its volume.

Ref. Rect. is parallel to A.O.R. $\Rightarrow$ shell method

$$V=\int_0^2 2\pi(x+2)(x^2-(-x^2))\,dx$$

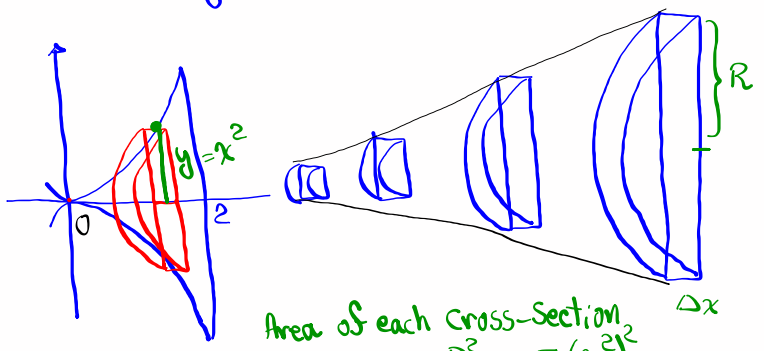
$$=2\pi\int_0^2 (x+2)\cdot 2x^2\,dx$$

$$=4\pi\int_0^2 (x^3+2x^2)\,dx$$

$$=\square$$

May 29-8:47 AM

3) Suppose we have cross-sections ⊥ to x-axis in the shape of semicircles with diameter on the region. Find its volume.



Area of each cross-section $\frac{\pi R^2}{2} = \frac{\pi (x^2)^2}{2}$

$$V = \int_0^2 \frac{\pi (x^2)^2}{2}\,dx = \frac{\pi}{2}\int_0^2 x^4\,dx = \frac{\pi}{2}\cdot\frac{x^5}{5}\Big|_0^2 = \frac{\pi}{10}\cdot 32$$

$$= \boxed{\frac{16\pi}{5}}$$

May 29-8:55 AM

Evaluate $\int_{-1}^{0} x^2\sqrt{x+1}\,dx$

$u = \sqrt{x+1}$

$u^2 = x+1 \rightarrow u^2 - 1 = x$

$2u\,du = dx$

$x = -1 \rightarrow u = 0$

$x = 0 \rightarrow u = 1$

$$= \int_0^1 (u^2-1)^2 \cdot u \cdot 2u\,du$$

$$= 2\int_0^1 u^2(u^4 - 2u^2 + 1)\,du$$

$$= 2\int_0^1 (u^6 - 2u^4 + u^2)\,du = 2\left[\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3}\right]\Big|_0^1$$

$$= 2\left[\frac{1}{7} - \frac{2}{5} + \frac{1}{3}\right] = \boxed{\quad}$$

May 29-9:03 AM

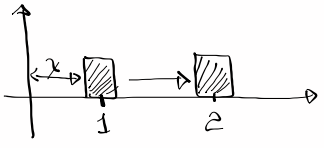
Introduction to work:

Work required to move an object from a to b is given by

$$W = \int_a^b f(x)\,dx$$

where $f(x)$ is the force applied to move the object.

An object is x units from the origin and we apply the force $f(x) = x^2 + 1$ to move it from 1 unit to 2 units from the origin.



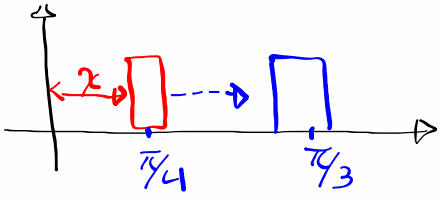
$$W = \int_1^2 (x^2+1)\,dx$$

$$= \left[\frac{x^3}{3} + x\right]\Big|_1^2 = \square$$

May 29-9:08 AM

An object is x feet from the origin. A force of $\sec^2 x$ pounds acts on it.

How much work is required to move it from $\frac{\pi}{4}$ to $\frac{\pi}{3}$.



$$W = \int_{\pi/4}^{\pi/3} \sec^2 x\,dx$$

$$= \tan x\Big|_{\pi/4}^{\pi/3}$$

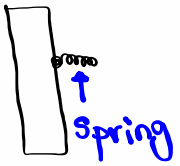
$$= \tan \pi/3 - \tan \pi/4$$

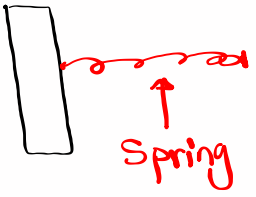
$$= (\sqrt{3} - 1) \text{ ft/lb.}$$

May 29-9:15 AM

Introduction to Hook's Law:

Suppose a force applied to a spring to stretch it.





Force $\rightarrow$ $f(x) = kx$

$k > 0$

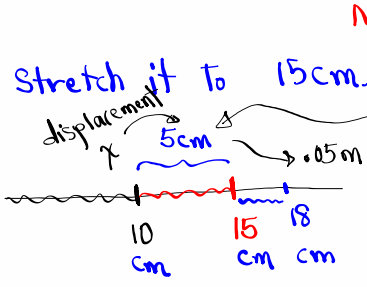
k is the spring constant

x cannot be too large.

May 29-9:19 AM

Suppose a spring has a length of 10 cm. A force of 40 N is applied to it to stretch it to 15 cm.

Newton



$F(x) = kx$

$40 = k \cdot (.05)$

$k = 800$

$f(x) = 800x$

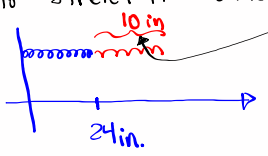
How much work is required to stretch it from 15cm to 18cm?

$$W = \int_{.05}^{.08} 800x\,dx = 800\,\frac{x^2}{2}\Big|_{.05}^{.08} = \square\ J.$$

May 29-9:23 AM

A spring has a length of 24 inches.

A force of 5 lb. is applied to it to stretch it 10 inches beyond original length.



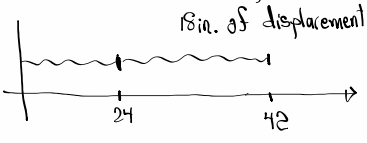
$f(x) = kx$

$5 = k \cdot 10$

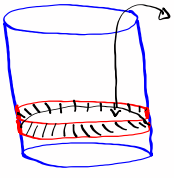
$k = \frac{1}{2}$

$f(x) = \frac{1}{2}x$

How much work is required to stretch it from natural length to 42 inches long.



$W = \int_0^{18} \frac{1}{2}x\,dx = \square$ in/lb.



Empty the water

You need a device strong enough to do the work.

May 29-9:31 AM